

Name:	Lab Section (circle): <b>am</b> <b>pm</b>	Score: <b>/ 125</b>
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For the purposes of this exam, please make  $g = 10 \text{ m/s}^2$ . That should simplify several calculations.

- A rough incline initially rises at an angle  $\theta_I = 53.13^\circ$  to the right. It supports a 200 g block ( $m_1$ ) attached by a rope and pulley to a 100 g block ( $m_2$ ) hanging freely on the right. The static coefficient of friction  $\mu_s$  is 0.5, and the kinetic coefficient  $\mu_k$  is 0.4. Presume that the larger block is sliding to the left down the slope and the lighter block is rising, both with an initial speed of  $v_0 = 0.5 \text{ m/s}$ . (a) Draw a free-body diagram, clearly showing all of the forces on each of the blocks. (b) Calculate the acceleration ( $a_I$ ) for each of the blocks. (c) How fast ( $v_I$ ) are they moving after 10 seconds have elapsed? (d) How far ( $d_I$ ) do they move in that 10 s? (e) Now consider a new angle of  $\theta_F = 36.87^\circ$ . Recalculate the acceleration ( $a_F$ ) using all the same initial conditions from part (a). (f) Now how fast ( $v_F$ ) is each block moving after 10 seconds? (g) How far ( $d_F$ ) do they move in that 10 s?
- You are driving in snowy weather, and the static coefficient of friction is  $\mu_s = 0.20$ . (a) What's the fastest speed  $v_0$  that you can go around an unbanked (level) corner in the snow if the curve has a radius of  $R = 50 \text{ m}$ . (b) At what angle  $\theta$  does your parking tag hang from your rearview mirror during this turn? (c) After you exit the curve, how much time  $t_s$  will it take for you to stop? Assume you are initially going the speed  $v_0$  calculated in part (a), and that you slow down without slipping. (d) After the brakes are applied, how far  $d$  does the car travel before coming to rest?
- (a) A book is lying flat on the floor. How much work  $W_0$  is needed to tilt a 0.20 kg book ( $m_b$ ) upright so that it is standing on its bottom edge? Assume the book is 23 cm tall, 15 cm wide and 3 cm thick, of uniform density, and symmetric. [Hint: where is the center of mass before and after it is tilted?] (b) How much work  $W_T$  would be required to fill a bookcase with five shelves, where the bottom of the first shelf is 15 cm off the ground and the other four shelves are spaced 25 cm one above the other? Assume each shelf holds 40 books and that each book starts by lying flat on the floor as in part (a). (c) If a book falls from the top shelf, what will be the kinetic energy  $K_{\max}$  and the speed  $v_{\max}$  when it hits the floor? Assume it hits the floor flat on its cover, not on its edge. Draw a sketch for each part of this question for full credit.
- A girl pushes a 3 kg block across a smooth floor with a steadily decreasing force  $F_x(t) = 24(t+1)^{-3/2}$ , where  $t$  is in seconds, and  $F_x$  is in Newtons. Assume that the block starts from rest at the origin at  $t = 0$ . (a) Find the acceleration  $a_x(t)$  and (b) evaluate  $a_x(t)$  at  $t = 7 \text{ s}$ . (c) Calculate the speed of the block  $v_x(t)$ , and (d) evaluate  $v_x(t)$  at  $t = 7 \text{ s}$ . (e) Calculate the position  $x(t)$ , and (f) find the distance traveled in the first 7 seconds. [Note:  $\int z^n dz = \frac{z^{(n+1)}}{(n+1)}$  for all  $n \neq -1$ .]
- A heavy cargo train ( $m_T = 5 \times 10^6 \text{ kg}$ ) travels through a mountain range and climbs a total elevation  $h = 1000 \text{ m}$  while traveling a distance of  $d = 50 \text{ km}$  (about a 2% grade, or  $1.146^\circ$ ). Its speed is constant at  $v_0 = 20 \text{ m/s}$ . There is a frictional force  $f$  that is 1.2% of its weight (independent of the angle of ascent). Calculate (a) the kinetic energy  $K_0$  of the train, (b) the change in the train's potential energy  $\Delta U$  as it rises to the summit, (c) the energy lost due to friction  $\Delta E_f$  during the climb, (d) the time  $t$  it takes to reach the summit, and (e) the average power output  $P_{\text{ave}}$  of the diesel engines during the trip.
- Extra Credit:** (a) Write out Hooke's Law (for a spring with constant  $k$  acting on a mass  $m$ ) in the form of a differential equation, using only the variables  $x$ ,  $t$ ,  $k$  and  $m$  and eliminating any reference to force or acceleration. If you let  $k/m = 1.0$  (for convenience), do you know of any functions  $x(t)$  which are a solution to your differential equation? (b) Suppose that instead of Hooke's Law for a spring, there was a weird force of the form  $F_x = +kx$ . What would be the corresponding differential equation for this system? Any ideas of what functions the solution for  $x(t)$  would have to look like?





Name: <i>FRANCESCO Meli</i>	Lab Section (circle): <b>am</b> <u>pm</u>	Score: <del>99</del> <b>125</b>
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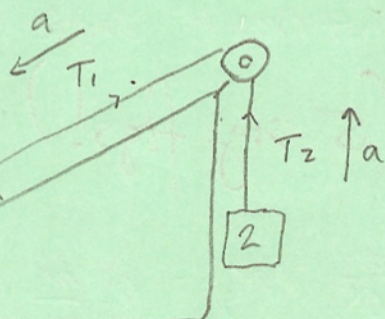
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2. You are driving in snowy weather, and the static coefficient of friction is  $\mu_s = 0.20$ . (a) What's the fastest that you can go around an unbanked (level) corner in the snow if the curve has a radius of 50 m. (b) At what angle does your parking tag hang from your rearview mirror during this turn? (c) After you exit the curve, how much time will it take for you to stop? Assume you are initially going the speed calculated in part (a), and that you slow down without slipping. (d) After the brakes are applied, how far does the car travel before coming to rest?
3. (a) How much work is needed to tilt a 0.20 kg book upright from lying flat on the floor so that it is standing on its edge? Assume the book is 23 cm tall, 15 cm wide and 3 cm thin, of uniform density, and symmetric. [Hint: where is the center of mass before and after it is tilted?] (b) How much work would be required to fill a set of four bookshelves, where the bottom of the first shelf is 15 cm off the ground and the other three shelves are spaced 25 cm one above the other? Assume each shelf holds 40 books and that each book starts by lying flat on the floor as in part (a). (c) If a book falls from the top shelf, what will be the kinetic energy and the speed when it hits the floor? Assume it hits the floor flat on its cover, not on its edge. Draw pictures for each part of this question for full credit.
4. A girl pushes a 3 kg block across a smooth floor with a steadily decreasing force  $F_x(t) = 24(t+1)^{-5/3}$ , where  $t$  is in seconds, and  $F_x$  is in Newtons. Assume that the block starts from rest at the origin at  $t = 0$ . (a) Find the acceleration  $a_x(t)$  and (b) evaluate  $a_x(t)$  at  $t = 7$  s. (c) Calculate the speed of the block  $v_x(t)$ , and (d) evaluate  $v_x(t)$  at  $t = 7$  s. (e) Calculate the position  $x(t)$ , and (f) find the distance traveled in the first 7 seconds.  
[Note:  $\int z^n dz = \frac{z^{n+1}}{(n+1)}$  for all  $n \neq -1$ .]
5. A heavy cargo train ( $5 \times 10^6$  kg) travels through a mountain range and climbs a total of 1000 meters while traveling a distance of 50 km (about a 2% grade, or  $1.146^\circ$ ). Its speed is constant at 20 m/s. There is a frictional force that is 1.2% of its weight (independent of the angle of ascent). Calculate (a) the kinetic energy of the train, (b) the change in the train's potential energy as it rises to the summit, (c) the energy lost due to friction during the climb, (d) the time it takes to reach the summit, and (e) the average power output of the diesel engines during the trip.
6. **Extra Credit:** (a) Write out Hooke's Law (for a spring with constant  $k$  acting on a mass  $m$ ) in the form of a differential equation, using only the variables  $x$ ,  $t$ ,  $k$  and  $m$  and eliminating any reference to force or acceleration. If you let  $k/m = 1.0$  (for convenience), do you know of any functions  $x(t)$  which are a solution to your differential equation? (b) Suppose that instead of Hooke's Law for a spring, there was a weird force of the form  $F_x = +kx$ . What would be the corresponding differential equation for this system? Any ideas of what functions the solution for  $x(t)$  would have to look like?



Problem



$$\alpha = 53.13^\circ$$

$$0.2 \text{ kg} = m_1 = 200 \text{ g}$$

$$0.1 \text{ kg} = m_2 = 100 \text{ g}$$

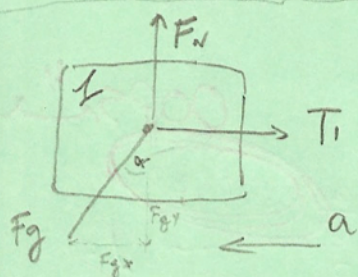
$$\mu_s = 0.5$$

$$\mu_k = 0.4$$

$$N_0 = 0.5 \frac{\text{m}}{\text{s}}$$

$$\theta = 36.87^\circ$$

(a)



$$\hat{x}: \sin(\alpha) F_g - T_1 = f = m_1 a$$

$$\hat{y}: F_N - \cos(\alpha) F_g = 0$$

$$\text{CON} \times \text{N}: f = F_N \mu_k \text{ (THE TWO BLOCKS ARE MOVING)}$$

$$\hat{x}: \checkmark$$

$$\hat{y}: T_2 - F_{g2} = m_2 a$$

$$T_2 = T_1$$

$$T_2 = m_2 a + m_2 g$$

$$\sin(\alpha) m_1 g - m_2 a - m_2 g - f = m_1 a$$

$$f = F_N \mu_k = \cos(\alpha) m_1 g \mu_k$$

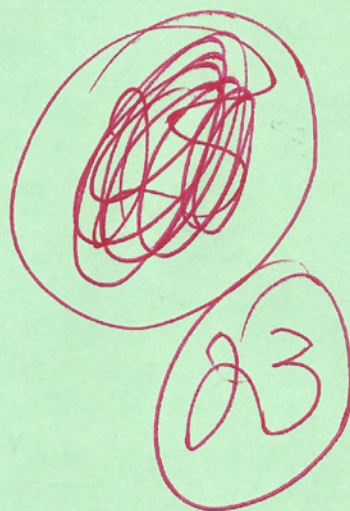
$$\sin(\alpha) m_1 g - m_2 a - m_2 g - \cos(\alpha) m_1 g \mu_k = m_1 a$$

$$\sin(\alpha) m_1 g - m_2 g - \cos(\alpha) m_1 g \mu_k = m_1 a + m_2 a = a(m_1 + m_2)$$

$$(b) a = \frac{\sin(\alpha) m_1 g - m_2 g - \cos(\alpha) m_1 g \mu_k}{m_1 + m_2} = 0.4 \frac{\text{m}}{\text{s}^2}$$

$$(c) N_f = N_0 + at = 0.5 \frac{\text{m}}{\text{s}} + 0.4 \frac{\text{m}}{\text{s}^2} \cdot 10 \text{ s} = 4.5 \frac{\text{m}}{\text{s}}$$

$$(d) x_f = x_0 + N_0 t + \frac{at^2}{2} = 0.5 \frac{\text{m}}{\text{s}} \cdot 10 \text{ s} + \frac{0.4 \frac{\text{m}}{\text{s}^2} \cdot 100 \text{ s}^2}{2} = 25 \text{ m}$$





(e) using formula (b) just with a different angle

$$a = -1.46 \frac{\text{m}}{\text{s}^2} \times = 1.46667 \text{ (5 sig figs!)}$$

$$(f) \cdot N_f = N_0 + at = -0.5 \frac{\text{m}}{\text{s}} + (-1.46 \frac{\text{m}}{\text{s}^2} \cdot 10\text{s}) = 0 \frac{\text{m}}{\text{s}} \quad (-14.1 \frac{\text{m}}{\text{s}})$$

$$(g) \quad t = \frac{N_f - N_0}{a} = \frac{0 - 0.5 \frac{\text{m}}{\text{s}}}{-1.46 \frac{\text{m}}{\text{s}^2}} = 0.342 \text{ s}$$

$$x_f = x_0 + N_0 t + \frac{at^2}{2} = 0.5 \frac{\text{m}}{\text{s}} \cdot 0.342 \text{ s} + \frac{(-1.46 \frac{\text{m}}{\text{s}^2}) (0.342^2 \text{ s}^2)}{2} =$$

$$= 0.171 \text{ m} - 0.0852 \text{ m} = 0.0852 \text{ m}$$

$$= 0.0852 \text{ m}$$

$$= 8.52 \text{ cm}$$

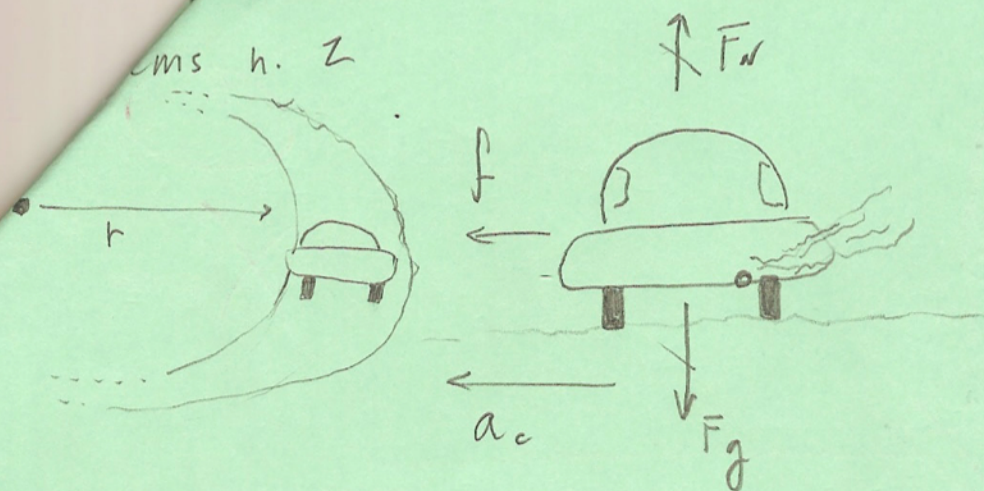
(you are about 2% off  
because of rounding)

Umm, your error is actually  
NOT rounding, but transcription!

you wrote 0.342 s for  $t$ , then used  
0.347 s! (Because your 7's & 2's are  
too similar for you to tell!)

I recommend crossing 7's so they are  
distinct.





$$\mu_s = 0.2$$

$$r = 50 \text{ m}$$

$\hat{x}$ :

$$f = ma$$

$$f = F_N \mu_s$$

$\hat{y}$ :

$$F_N = F_g = mg$$

connectors:

$$a_c = \frac{v^2}{r}$$

21

$$(1) f = mg \mu_s$$

$$(2) ma = mg \mu_s$$

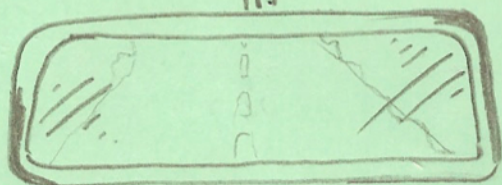
$$(3) a = \frac{mg \mu_s}{m} = g \mu_s$$

so

$$\frac{v^2}{r} = g \mu_s \quad [\text{from (3) \& connector}]$$

$$v = \sqrt{g \mu_s r} = \sqrt{10 \frac{\text{m}}{\text{s}^2} \cdot 0.2 \cdot 50 \text{ m}} = 10 \frac{\text{m}}{\text{s}} = 36 \frac{\text{km}}{\text{h}} = \frac{22.5 \text{ miles}}{\text{h}} \quad (a)$$





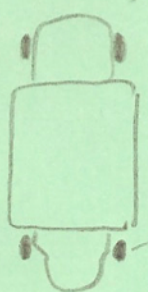
PARK  
084

PARKING  
TAG

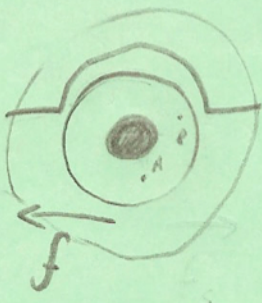
$\theta = ?$

X

a ↓



↓  $f_s$



← Slowing Down

$$(c) -f_s = ma_{max}$$

$$-mg\mu_s = ma_{max}$$

$$a = g\mu_s$$

$$= -10 \frac{m}{s^2} \cdot 0.2 = -2 \frac{m}{s^2}$$

$$v_f = v_0 + at$$

$$t = \frac{v_f - v_0}{a} = \frac{0 - \frac{10 m}{s}}{-2 \frac{m}{s^2}} = \boxed{5 \text{ sec}}$$

(d)

$$v_f^2 = v_0^2 + 2ax$$

$$x = \frac{v_f^2 - v_0^2}{2a} = \frac{0 - 100 \frac{m^2}{s^2}}{2(-2 \frac{m}{s^2})} = \frac{-100 m}{-4} = \boxed{25 m}$$



le m n.3

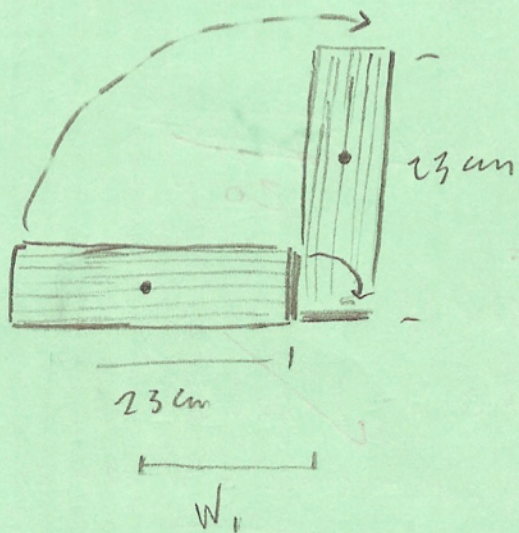
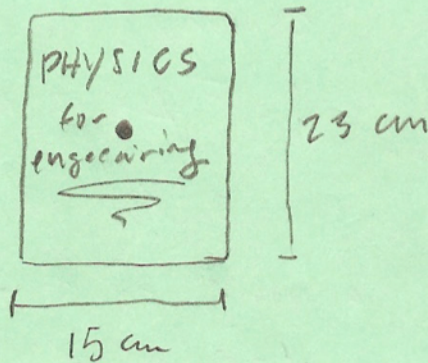
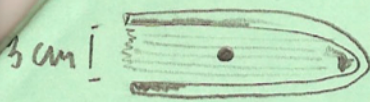
$$0.23 \text{ m} = h = 23 \text{ cm}$$

$$0.15 \text{ m} = w = 15 \text{ cm}$$

$$0.03 \text{ m} = t = 3 \text{ cm}$$

$$m = 0.2 \text{ Kg}$$

20



$W_1$  is 0 because you don't need to apply any force to slide the book.

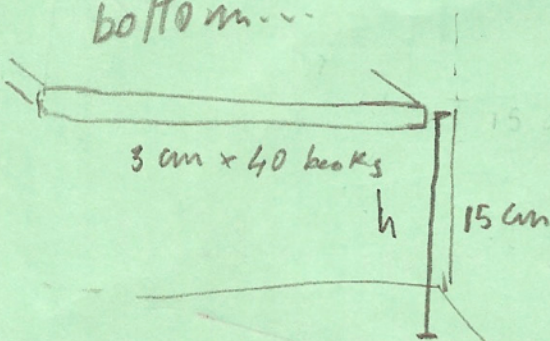
$$\begin{aligned} (a) W_2 &= F \cdot \Delta y = m \cdot g \cdot \Delta y = \\ &= 0.2 \text{ Kg} \cdot 10 \frac{\text{m}}{\text{s}^2} \cdot \left( \frac{0.23}{2} - \frac{0.03}{2} \right) = \\ &= 0.2 \text{ J} \end{aligned}$$

10 kind of shelf.

In English, it's 1<sup>st</sup>, not 1<sup>o</sup> how we need to consider the work done by raising the book

- height necessary to make the book feet in the shelf (considering already the book standing).

bottom...



$$15 \text{ cm} + \frac{23}{2} = 26.5 \text{ cm} = 0.265 \text{ m}$$

$$W_3 = 0.2 \text{ Kg} \cdot 10 \frac{\text{m}}{\text{s}^2} \cdot \Delta y = 0.53 \text{ J} \rightarrow 0.3 \text{ J}$$

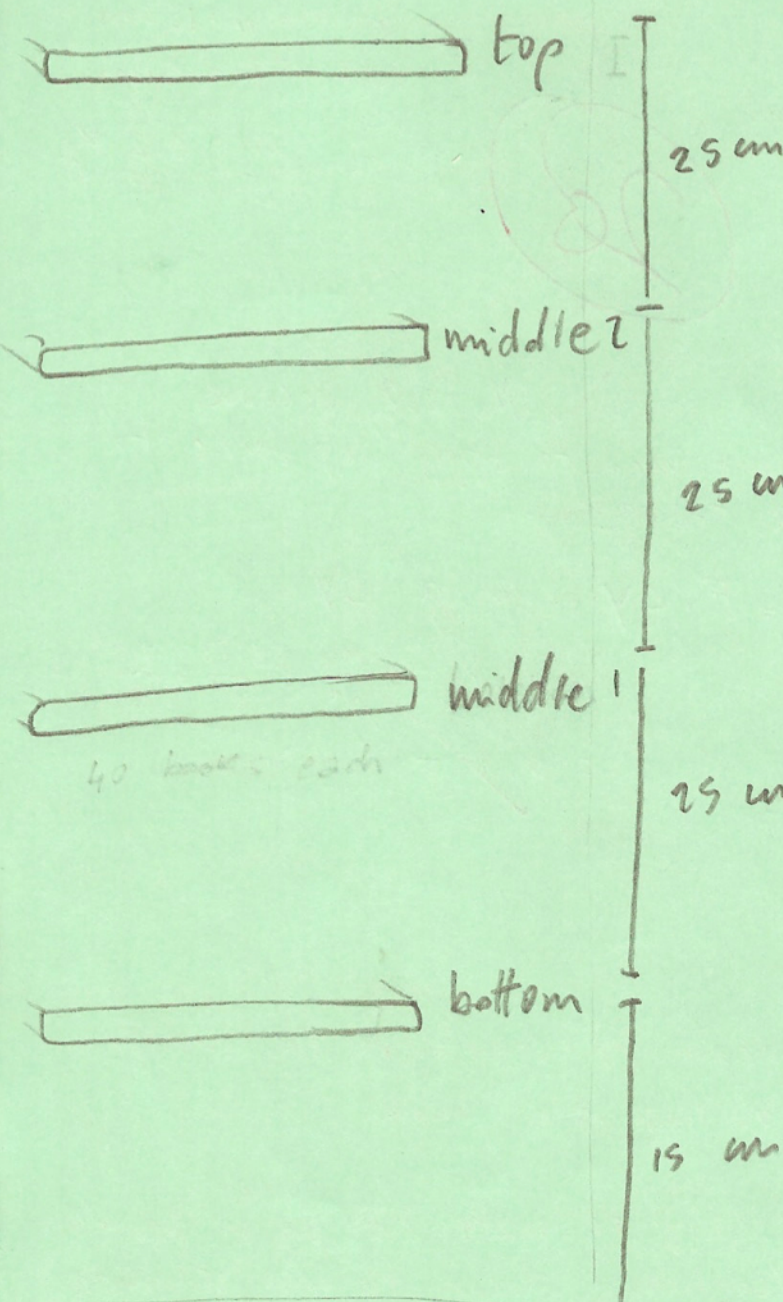
- total work to insert 1 book in the first shelf  $\Rightarrow 0.3$

$$W_2 + W_3 = 0.2 \text{ J} + 0.3 \text{ J} = 0.5 \text{ J}$$

- total work 1<sup>st</sup> shelf =

$$3.05 \cdot 40 \text{ books} = 122 \text{ J} \rightarrow 20 \text{ J}$$





- work to raise a book 25 cm.  
 $W = F \Delta x = mg$   
 $= 0.2 \text{ Kg} \cdot 10 \frac{\text{m}}{\text{s}^2} \cdot 0.25 \text{ m} = 0.5 \text{ J}$
- work to slide the book in  
 $W = 0$  (we are not against g.)

25 cm •  $W$  <sup>1st</sup> shelf = ~~29.2 J~~  
20

25 cm •  $W$  <sup>2nd</sup> shelf  
 $(0.73 \text{ J} + 0.5 \text{ J}) \cdot 40 \text{ books} =$   
 (work till first shelf + W for 25 cm)  
 • 40 books = ~~49.2 J~~  
40

15 cm •  $W$  <sup>3rd</sup> shelf  
 $W$  2nd shelf + ( $W$  15 cm raise • 40 books)  
 $49.2 + (0.5 \text{ J} \cdot 40) =$  ~~69.2 J~~  
60

even if there is a pattern...

BIG TOTAL =  
 $W$  1st shelf +  $W$  2nd shelf +  $W$  3rd shelf +  $W$  4th shelf =  
 $29.2 + 49.2 + 69.2 + 89.2 =$   
 (b) ~~236.8 J~~

200 exactly.

•  $W$  <sup>4th</sup> shelf  
 $W$  3rd shelf + ( $W$  15 • 40 books) =  
 $69.2 + (0.1 \cdot 40) =$  ~~89.2 J~~  
80

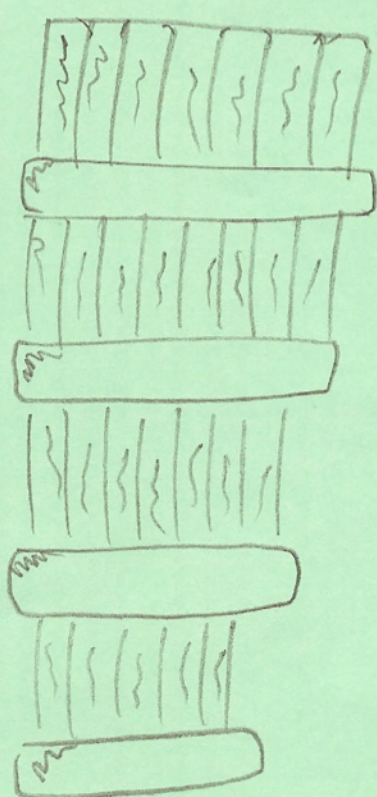


five problem n. 3

total height in between the floor and the center of the book.

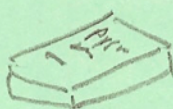
$$25 + 25 + 25 + 15 + \left( \frac{23}{2} - \frac{3}{2} \right) = 100 \text{ cm} = \boxed{1 \text{ m}}$$

$$K_f = M_o = mgh = 0.2 \text{ Kg} \cdot 10 \frac{\text{m}}{\text{s}^2} \cdot 1 \text{ m} = 20 \text{ J}$$



$$K_o = 0$$

$$M_o = mgh$$



$$K_f = M_o$$

$$M_f = 0$$

$$J = ?$$



Problem 4

$$F_x(t) = 24(t+1)^{-\frac{5}{3}}$$

$$m = 3 \text{ Kg}$$

$$t_0 = 0$$

$$(a) a_x(t) = \frac{24(t+1)^{-\frac{5}{3}}}{3} = \boxed{8(t+1)^{-\frac{5}{3}}} \quad \checkmark$$

$$(b) a_x(7) = 8 \cdot 8^{-\frac{5}{3}} = \frac{8}{8^{\frac{5}{3}}} = \frac{8}{\sqrt[3]{8^5}} = 0.25 \frac{\text{m}}{\text{s}^2} \Rightarrow \boxed{\cancel{0.25} \frac{\text{m}}{\text{s}^2}} \quad \checkmark$$

$$(c) \int_0^t 8(t+1)^{-\frac{5}{3}} dt = (8) \cdot \left(-\frac{3}{2}\right) (t+1)^{-\frac{2}{3}} = \boxed{-12(t+1)^{-\frac{2}{3}} + C = v_x(t)} \quad \checkmark$$

$$(d) v_x(7) = -12(8)^{-\frac{2}{3}} = \boxed{-3 \frac{\text{m}}{\text{s}}} = \boxed{3 \frac{\text{m}}{\text{s}}} \quad \checkmark$$

$$(e) x(t) = \int v_x(t) dt = \int -12(t+1)^{-\frac{2}{3}} = \frac{-12(t+1)^{\frac{1}{3}}}{\frac{1}{3}} = 3 \cdot (-12)(t+1)^{\frac{1}{3}} =$$

$$\boxed{-36(t+1)^{\frac{1}{3}} + C = x(t)} \quad \checkmark \quad C = +36$$

$$(f) \Rightarrow -36(8)^{\frac{1}{3}} = \boxed{-72 \text{ m}} = \boxed{72 \text{ m}} \quad \checkmark$$

No  $\rightarrow$  preserve  $\leftarrow \rightarrow$ !

~~12~~ 18

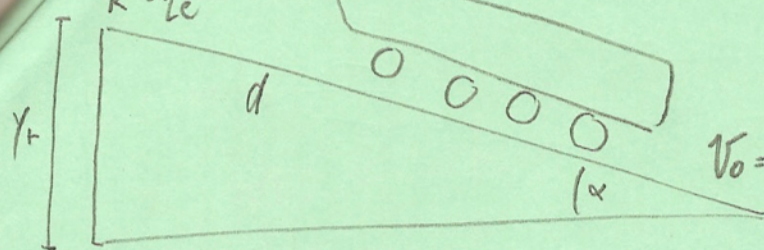
$C \neq 0 \rightarrow C = +12$   
here!



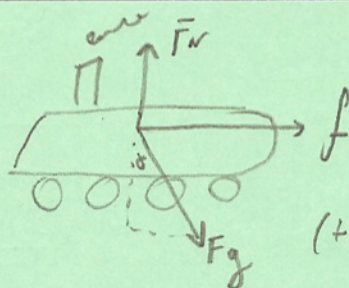
# Problem 5

$$v_f = 5e^{11}$$

$$k = 2e^9$$



$$v_0 = 0 \quad k = 2e^9$$



(the angle is not proportional)

$$m = 5 \cdot 10^6 \text{ kg}$$

$$d = 50 \text{ km}$$

$$\alpha = 1.146^\circ$$

$$v = 20 \frac{\text{m}}{\text{s}}$$

$$h_f = 1000 \text{ m}$$

$$f = 600.000 \text{ N}$$

$$f = 1.2\% \text{ of weight } F_{\text{train}} = \frac{5 \cdot 10^6 \cdot 10}{100} \cdot 1.2 = 600.000 \text{ N}$$

$$(a) \Rightarrow K = \frac{1}{2} m v^2 = \left( \frac{1}{2} \right) 5 \cdot 10^6 \text{ kg} \cdot 20^2 \frac{\text{m}^2}{\text{s}^2} = 1e^9$$

$$(b) \Rightarrow M = m g h_f - m g h_i = 5 \cdot 10^6 \text{ kg} \cdot 10 \frac{\text{m}}{\text{s}^2} \cdot 1000 \text{ m} = 5e^{10} (!)$$

$$(c) W_f = -f \Delta x = -600.000 \text{ N} \cdot 50.000 \text{ m} = -3e^{10}$$

ENERGY LOST =  $3e^{10} \text{ J}$  (CAUSE OF FRICTION)

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$$(d) t = \frac{d}{v} = \frac{50.000 \text{ m}}{20 \frac{\text{m}}{\text{s}}} = 2500 \text{ s} \approx 41.66 \text{ min} = 0.694 \text{ hours}$$

$$(e) M_0 + K_0 + W_f = M_f + K_f \rightarrow M_f - W_f = M_c^* = 4.7e^{10}$$

$$P = F \cdot v = \frac{W}{\Delta x} \cdot v = \frac{-M_d}{\Delta x} \cdot v = \frac{-4.7e^{10}}{50000} = 1.88e^6 \text{ W}$$

$M_c$  = potential energy without the energy spent to go against the friction.