Physics 195

Midterm 3, Summer 2010

D. Brownell

Name:	Lab Section: **AM PM**	Score: **/ 125**

There are six problems on this exam, each of which is equally weighted. You may use your calculator plus a single equation sheet handwritten on one side. If you have forgotten either of these tools, shame on you ☺ (I might have spares – come ask). Write your answers only on the colored paper provided for you, **one question per sheet**. Do not put multiple questions on one sheet. You may use the back of a sheet only if you need more room to work out the question on the front.

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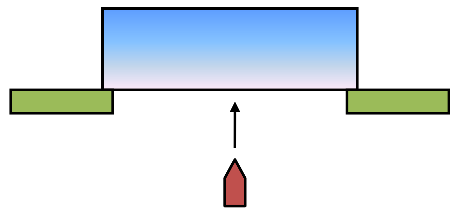
For the purposes of this exam, please make $\mathbf{g} = 10\text{ m/s}^2$. That should simplify several calculations.

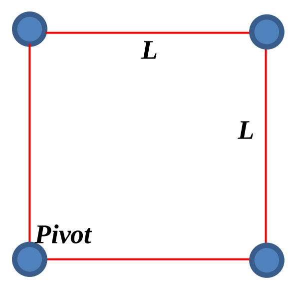
1. A 10-g bullet moving 200-m/s strikes and passes through a 250-g block initially at rest, as shown. The bullet emerges from the block with a speed of 100-m/s. (a) What momentum Δp was transferred to the block? (b) To what maximum height h will the block rise above its initial position? (c) If the block is 15-cm thick, how long t_0 is the bullet inside the block (assuming constant deceleration)? (d) In words, describe for me what will happen if the bullet hits the block all the way on the left side instead of at the center. Do you think that the center of mass of the block will rise as far as it did in part (b) above? You should **not** resort to any calculation for this part.



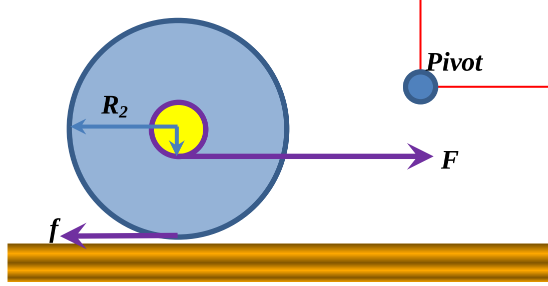
2. Two blocks with masses m_1 = 2-kg and m_2 = 3-kg are placed on a horizontal frictionless surface. A light spring is placed in a horizontal position between the blocks. The blocks are pushed together, compressing the spring, and then released from rest. (a) After the blocks lose contact with the spring, the 3-kg mass has a speed of v_2 = 2-m/s (along the positive x-axis). Using momentum only, what is the velocity v_1 the other block? (b) How much potential energy U_S was stored in the spring before the blocks were released? (c) What is the spring constant k if the spring was compressed 10 cm from its equilibrium?

3. A regulation basketball has a mass M = 0.56-kg and a diameter D = 25-cm. It may be approximated as a thin spherical shell with a moment of inertia 2/3 MR^2. The block starts from rest and rolls down a 36.87° incline. (a) Using energy, what is the final speed v_f after it rolls without slipping for a distance d = 4-m? (b) How long t_f does it take to roll down? (c) What fraction of its total kinetic energy K_T is its rotational kinetic energy K_R?

4. The rigid object shown lies in the plane of the page and is rotated about an axis perpendicular to the paper and through the pivot point P which passes through one of the vertices of the object. It is made from four identical point masses with M_0 = 1-kg. (a) If L = 0.5-m, what is the total moment of inertia I_T of the object? (b) The total kinetic energy K_R of the object as it rotates is 18 J. What are the linear speeds of each of the four objects? Neglect the mass of the connecting rods.



5. A Yo-Yo of mass M is pulled along the floor by a force F, and it rolls without slipping starting from rest. The inner radius R_1 is ¼ the outer radius R_2. For the purposes of this calculation you may presume that it is a uniform disk of radius R_2. (a) What is the linear acceleration a of the Yo-Yo across the floor? (b) What is the minimum value of the coefficient of static friction μ_S in order to maintain this acceleration?



6. A thin rod has a non-uniform density described by $\lambda(x) = 2\cos(2\pi x)$. The rod is 50 cm long, and has a pivot point at its geometrical center. (a) What is the total mass of the rod? (b) Where is the rod's center of mass? Use symmetry arguments to justify your calculation. (c) What is the moment of inertia of the rod? For this answer, set up the integral but do not evaluate. (It requires several rounds of integration by parts to complete…)

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(1) >>>>>>>>>>>>>>>>>>>>>>>>>

$\Delta p_{block} = \Delta p_{bullet} = p_f - p_0 = m\Delta v = \boxed{1.0\ \text{kg}\cdot\text{m/s}}$

$v_{0block} = p_{block}/m = \boxed{4\ \text{m/s}}$

5 pts

$v_0^2 = 2gh \rightarrow h_{block} = \dfrac{v_0^2}{2g} = \boxed{0.8\ \text{m}}$

$\Delta y = \bar{v}\cdot t = 0.5(v_0 + v_f) \rightarrow t = \dfrac{2\Delta y}{\bar{v}} = \boxed{0.001\ \text{s} = 1\ \text{ms}}$

If struck on the left side, the block will rise, but it will also be spinning clockwise. Since some of its momentum will be given to the rotational motion, it won't rise as high.

(2) >>>>>>>>>>>>>>>>>>>>>>>>>

$p_1 = -p_2 \rightarrow m_1 v_1 = -m_2 v_2 \rightarrow \boxed{v_1 = -3\ \text{m/s (left, } -\hat{x} \text{ direction)}}$

$K_0 = 0,\ U_0 = \tfrac{1}{2}kx^2,\ K_F = \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2,\ U_F = 0$

$U_0 = K_F = \boxed{15\ \text{J}}$

8 ea

$k = \dfrac{2U_F}{x^2} = \boxed{3000\ \text{N/m}}$

(3) >>>>>>>>>>>>>>>>>>>>>>>>>

$\boxed{U_0 = mgh = mgd\sin\theta = 13.44\ \text{J} = K_F} = K_L + K_R$

$K_F = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}(\tfrac{2}{3}mr^2)(v/r)^2 = \boxed{\tfrac{5}{6}mv_F^2}$

$v_F = \sqrt{\dfrac{6K_F}{5m}} = \sqrt{\dfrac{6U_0}{5m}} = \boxed{5.36\ \text{m/s}}$

5 ea

$t = \dfrac{2d}{v_F} = \boxed{1.49\ \text{s}}$

$\dfrac{K_R}{K_T} = \dfrac{2}{5} = 0.4 \rightarrow \boxed{K_R = 40\%\ \text{of}\ K_T}$

(4) >>>>>>>>>>>>>>>>>>>>>>>>>

$I = \sum m_i r_i^2 = 0 + 2\cdot ML^2 + M(\sqrt{2}L)^2 = 4ML^2 = \boxed{1.0\ \text{kg}\cdot\text{m}^2}$

$K = \tfrac{1}{2}I\omega^2 \rightarrow \omega = \sqrt{\dfrac{2K}{I}} = \boxed{6\ \text{rad/s}}$

8 ea

$\boxed{v_1 = 0\ \text{(pivot)},\ v_2 = v_4 = \omega L = 3\ \text{m/s, and}\ v_3 = \omega\sqrt{2}L = 4.24\ \text{m/s}}$

(5) >>>>>>>>>>>>>>>>>>>>>>>>>

$\sum F_y:\ F_N = mg$

$\sum F_x:\ F - f = ma$

$\sum \tau:\ fR_2 - FR_1 = I\alpha$

15 pts

$I = \tfrac{1}{2}MR_2^2,\ \text{and}\ \alpha = a/R_2$

Using torques $\rightarrow f = \tfrac{1}{2}Ma + \tfrac{1}{4}F$

Plugging into $F_H \rightarrow \boxed{a = \dfrac{F}{2M}}$

$f = \tfrac{1}{2}M\left(\dfrac{F}{2M}\right) + \tfrac{1}{4}F = \mu_s F_N = \mu_s Mg$

$\boxed{\mu_s = \dfrac{F}{2Mg}}$

10 pts

(6) >>>>>>>>>>>>>>>>>>>>>>>>>

8 pts

$dm = \lambda(x)dx = 2\cos(2\pi x)dx$

$M = \int dm = \int_{-0.25}^{0.25} 2\cos(2\pi x)dx = \dfrac{\cancel{2}\sin(2\pi x)}{\cancel{2}\pi}\Bigg|_{-0.25}^{0.25} = \boxed{\dfrac{2}{\pi} = 0.637\ \text{kg}}$

$X_{CM} = \dfrac{1}{M}\int x\,dm = \dfrac{1}{M}\int_{-0.25}^{0.25} 2x\cos(2\pi x)dx \rightarrow$ Integrate by parts:

$u = x,\ du = dx;\ dv = 2\cos(2\pi x)dx,\ v = \dfrac{1}{\pi}\sin(2\pi x)$. Then:

$\int u\,dv = uv - \int v\,du = \dfrac{x}{\pi}\sin(2\pi x) - \dfrac{2}{\pi}\int \sin(2\pi x)dx$

12 pts

$= \dfrac{x}{\pi}\sin(2\pi x) + \dfrac{2}{\pi}\cos(2\pi x)\Bigg|_{-0.25}^{0.25} = \boxed{0 = X_{CM}}$

$\boxed{I = \int x^2 dm = \int_{-0.25}^{0.25} 2x^2\cos(2\pi x)dx} \rightarrow$ Integrate by parts, Twice!

$I = \left(\dfrac{x}{\pi^2}\cos(2\pi x) + \left(\dfrac{x^2}{\pi} - \dfrac{1}{2\pi^3}\right)\sin(2\pi x)\right)\Bigg|_{-0.25}^{0.25}$

$\boxed{I = 0.00754\ \text{kg}\cdot\text{m}^2}$

5 pts + 5 xc

$wr = v$

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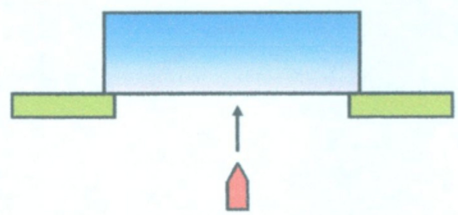

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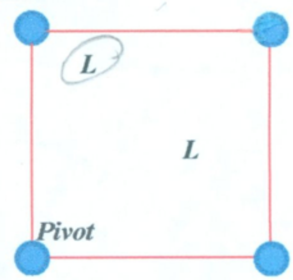
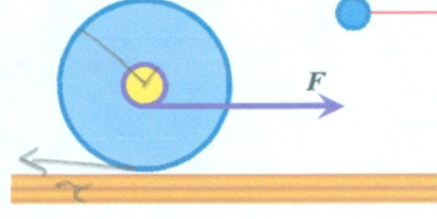
3. A regulation basketball has a mass $M = 0.56$-kg and a diameter $D = 25$-cm. It may be approximated as a thin spherical shell with a moment of inertia 2/3 MR^2. (a) Starting from rest, what will be the final speed after it rolled without slipping for a distance of 4-m down an incline at 36.87° below the horizontal? (b) How long did it take to roll down? (c) What fraction of its total kinetic energy K_T was in the form of rotational kinetic energy K_R about the CM?

4. The rigid object shown lies in the plane of the page and is rotated about an axis perpendicular to the paper and through the pivot point P which passes through one of the vertices of the object. It is made from four identical point masses with $M_0 = 0.1$ kg. (a) If $L = 0.50$ m, what is the total moment of inertia I_T of the object? (b) The total kinetic energy K_R of the object as it rotates is 18 J. What are the linear speeds of each of the four objects? Neglect the mass of the connecting rods.



5. A Yo-Yo is pulled along the floor starting from rest by a force F, and it rolls without slipping. The inner radius R_1 is ¼ the outer radius R_2. (a) What is the rate of linear acceleration a of the Yo-Yo along the floor? (Hint: evaluate the torque about the point of contact with the floor) (b) What is the minimum value of the coefficient of static friction μ_S in order to maintain this acceleration?

$m_{2\pi} = M$



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Problem 1

$0.01\,Kg = m_1 = 10g$

$v_1 = 200\,\frac{m}{s}$

$0.25\,Kg = m_2 = 250\,g$

$v_2 = 0\,\frac{m}{s}$

$v_{f1} = 100\,\frac{m}{s}$

Thick $= Th = 0.15\,m$

(a) $p_1 + p_2 = p_{f1} + p_{f2}$ (ELASTIC) $=$

$= m_1 v_1 + m_2 v_2 = m_1 v_{f1} + m_2 v_{f2}$

$m_2 v_{f2} - \cancel{m_2 v_2}^{\,0} = m_1 v_1 - m_1 v_{f1} =$

$\Delta p = m_1 (v_1 - v_{f1}) = 0.01\,Kg\,(200 - 100)\,\frac{m}{s} = \boxed{1\,Kg\,\frac{m}{s}}$ (a)

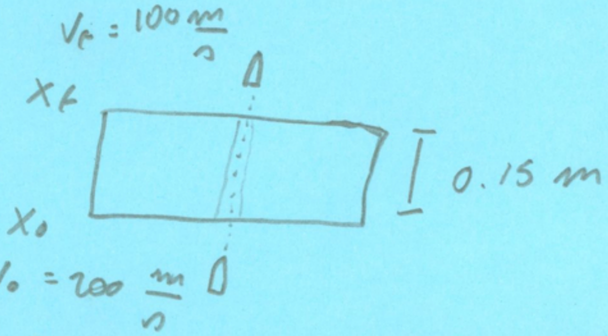
(b) $v_{f1} = \frac{p_{f2}}{m_2} = \frac{1\,Kg\,\frac{m}{s}}{0.25\,Kg} = 4\,\frac{m}{s}$

$K = \frac{m_2 v^2}{2} = \frac{0.25\,Kg \cdot 4^2\,\frac{m^2}{s^2}}{2} = 2\,Kg\,\frac{m^2}{s^2}$

$\cancel{U_0}^{\,0} + K_0 = U_f + \cancel{K_f}^{\,0}$ $\qquad \frac{mv^2}{2} = m_2 g h = 2\,Kg\,\frac{m^2}{s^2}$

$h = \frac{2\,Kg\,\frac{m^2}{s^2}}{m_2 g} = \frac{2\,Kg\,\frac{m^2}{s^2}}{0.25\,Kg \cdot 10\,\frac{m}{s^2}} = \boxed{0.8\,m}$ (b)

(c) ~~$v_f = v_0 + at$~~



$a = \frac{v_f - v_0}{t}$ [2]

$x_f = \cancel{x_0}^{\,0} + v_0 t + \frac{at^2}{2}$

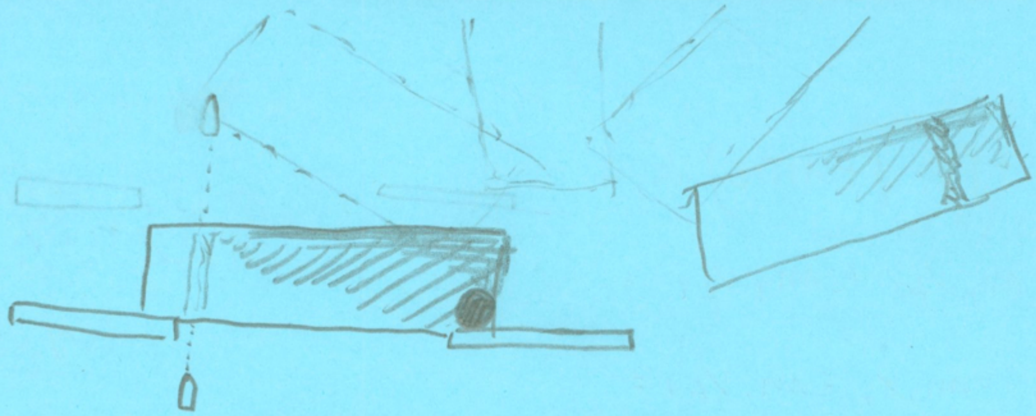
$x_f = v_0 t + \frac{(v_f - v_0)\,t^2}{2t} = v_0 t + \frac{v_f t}{2} - \frac{v_0 t}{2} = t\left(v_0 + \frac{v_f}{2} - \frac{v_0}{2}\right) = t\left(\frac{v_0}{2} + \frac{v_f}{2}\right)$

$t = \frac{x_f}{\left(\frac{v_0}{2} + \frac{v_f}{2}\right)} = \frac{0.15\,m}{\frac{200}{2}\frac{m}{s} + \frac{100}{2}\frac{m}{s}} = \frac{0.15}{(100 + 50)\,\frac{1}{s}} = \boxed{0.\cancel{01}\ sec.}$ (c)

0.001 sec.

23

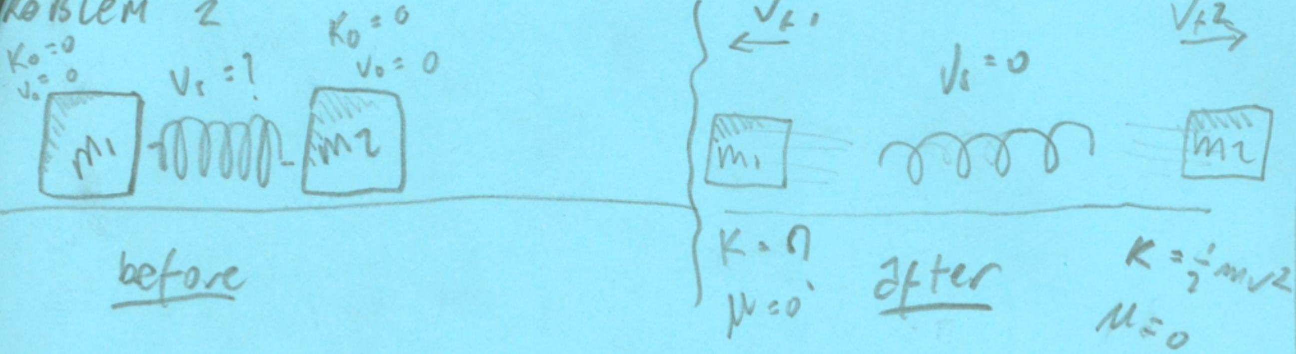
(d)



the block will flip over on the left as shown in the figure. the maximum haight is riched when the bullet hits the block right on the center of the mass. in other cases such as this one, The ~~b~~ block will move up and will flip clockwise because the pivot on the system (◎) im on the RIGHT HAND SIDE.

will it go higher, or NOT as high?
(compared with part b...)

PROBLEM 2



$m_1 = 2\,Kg$

$m_2 = 3\,Kg$

$v_{f1} = ?$

$v_{f2} = 2\,\frac{m}{s}$

$U_{s0} + \cancel{U_{sf}} = \cancel{K_0} + K_f$

$U_{s0} = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2$ [2]

$m_1 \tilde{v}_1 + m_2 \tilde{v}_2 = m_1 \tilde{v}_{1f} + m_2 \tilde{v}_{2f}$ [1]

$v_{1f} = \dfrac{\cancel{m_1 v_1} + \cancel{m_2 v_2} - m_2 v_{2f}}{m_1} = \dfrac{-3\,Kg \cdot 2\,\frac{m}{s}}{2\,Kg} = -3\,\frac{m}{s}$ (a)

20

$U_{s0} = \frac{1}{2} \cdot 2\,Kg \cdot \left(-3\,\frac{m}{s}\right)^2 + \frac{1}{2}\, 3\,Kg \left(2\,\frac{m}{s}\right)^2 =$

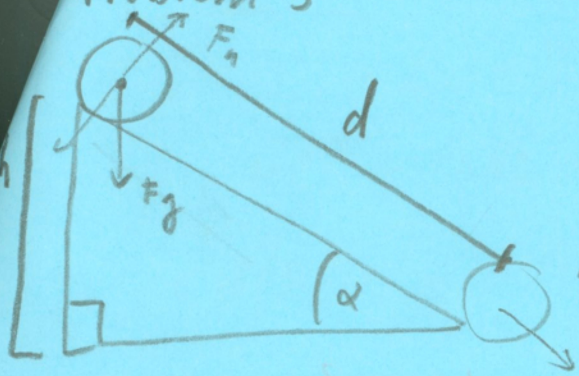
$9\,\frac{m^2}{s^2} \cdot Kg + 6\,\frac{m^2}{s^2}\,Kg = \boxed{15\,J}$ (b)

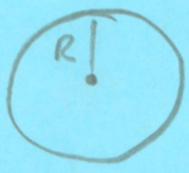
$U_s = \frac{1}{2} K x^2$

$K = \dfrac{U_s}{x^2} = \dfrac{2 \cdot 15\,\frac{m^2}{s^2}\,Kg}{0.01^2\,m^2} = \boxed{\cancel{15000}\,\frac{Kg}{s^2}}$ (c) $=$

$= 3000\,\frac{N}{m}.$

Problem 3





$\alpha = 36.87°$

$D = 0.25\ m$

$R = 0.125\ m$ $d = 4\ m$

$M = 0.56\ Kg$

$I = \frac{2}{3} MR^2$

15

$M_f = 0$

$K = \frac{mv^2}{2}$

(a) $\alpha = \frac{F \cdot R}{I} = \frac{\sin(36.87)\, \cancel{m} g \cdot 0.125\ m}{\cancel{\frac{2}{3} M} R^2} = \frac{\sin(36.87)\, 10 \frac{m}{s^2} \cdot 0.125\ m}{\frac{\cancel{2}\, 5}{3} (0.125)^2} =$

$= \cancel{72}\ 28.8\ \frac{rad}{s^2}$

Now, pivot is at ground, so $I = I_{cm} + MD^2 = \frac{5}{3} mR^2$

$w_f^2 = \cancel{w_0^2}\, 0 + 2\alpha(\theta_f - \cancel{\theta}\, 0) = 2\alpha\left(\frac{4\ m}{0.125\ m}\right) = 2 \cdot 72\ \frac{rad}{sec^2} \cdot 32\ rad = \cancel{4608}\ 1843.2\ \frac{rad}{s^2}$

$w_f = \cancel{67.88}\ 42.93\ \frac{rad}{sec} \longrightarrow \boxed{v = \omega r = 5.36\ m/s}$

$h = \sin(36.87)\, 4\ m = 2.4\ m$

$M_0 + \cancel{K_0} = \cancel{M_f} + K_f = \frac{1}{2} m v_f^2 + \frac{1}{2} I W^2$

$mgh = \frac{1}{2} m v_f^2 + \frac{1}{2} I W^2$

$I = \frac{2}{3} MR^2 = \frac{2}{3}\, 0.56\ kg \cdot (0.125)^2\ m^2 = 0.0058\ m^2\ Kg$

$v_f = \sqrt{\cancel{\frac{\left[mgh - \frac{1}{2} I W^2\right] 2}{m}}} = \sqrt{\frac{(13.44 - 13.36)\, 2}{0.56}} = \boxed{0.277\ \frac{m}{s}}$ (a)

(c) $K_{TOT} = \frac{1}{2} m v^2 + \frac{1}{2} I w^2$

$\frac{1}{2} m v^2 = \frac{1}{2} 0.56 \, kg \cdot 0.277^2 \frac{m^2}{s^2} = 0.214$

$\frac{1}{2} I w^2 = \frac{1}{2} M R^2 w^2 = 13.36$

$TOT = 0.214 + 13.36 = 13.38$

$\frac{13.38}{100} = \frac{13.36}{x}$

$x = \frac{13.36}{13.38} \cdot 100 = 99.85\% = \boxed{\frac{9985}{10000}}$ (C)

$K_L = \frac{1}{2} m v^2$

$K_R = \frac{1}{2} I \omega^2 = \frac{1}{3} m v^2$

$K_T = \frac{5}{6} m v^2$

$\frac{K_R}{K_T} = \frac{(\frac{1}{3})}{(\frac{5}{6})} = \frac{6}{15} = 0.4 = 40\%$

(b) $v_0 = 0$

$v_f = 0.277 \frac{m}{s}$

$x_0 = 0$

$x_f = 4 \, m$

$v_f = v_0 + at \qquad a = \frac{v_f - \cancel{v_0}}{t}$

$x_f = \cancel{x_0} + \cancel{v_0 t} + \frac{at^2}{2} = \frac{\frac{v_f - \cancel{v_0}}{\cancel{(t)}} t^2}{2} = \frac{v_f t}{2}$

$t = \frac{2x_f}{v_f} = \frac{2 \cdot 4 \, m}{0.277 \frac{m}{s}} = 28.88$

yey! Great answer!

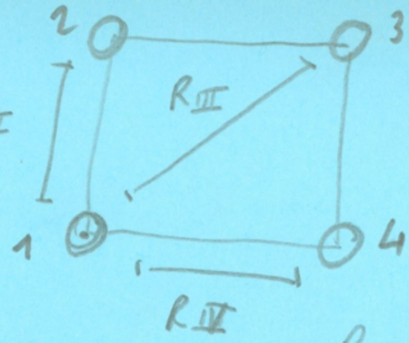
... sike ...

roblem 4

$$I_T = I_1 + I_2 + I_3 + I_4$$

$$= MR^2 + MR_{II}^2 + MR_{III}^2 + MR_{IV}^2$$

$$= M(\cancel{R_I^2} + R_{II}^2 + R_{III}^2 + R_{IV}^2)$$



$R_I = 0$ $\quad R_{II} = R_{IV}$

$$= 1 Kg (0.5^2 m^2 + 0.707^2 m^2 + 0.5^2 m^2) =$$

$$= \boxed{1 Kg \cdot 1 m^2} \text{ (a)}$$

25

$$K = \frac{I w^2}{2}$$

$$w = \sqrt{\frac{K \cdot 2}{I}} = \sqrt{\frac{18 Kg \frac{m^2}{s^2} \cdot 2}{1 Kg \cdot 1 m^2}} = \boxed{6 \frac{rad}{s}}$$

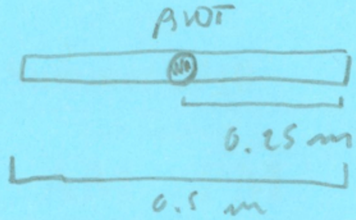
the angular velocity is suppose to be the same. (expect for 1. it doesn't move)

$$V_4 = V_2 = W \cdot R_{II} = 6 \frac{rad}{sec} \cdot 0.5 m = 3 \frac{m}{s} \text{ (b)}$$

$$V_3 = W \cdot R_{III} = 6 \frac{rad}{sec} \cdot 0.707 m = 4.2 \frac{m}{s} \text{ (b)}$$

Problem 6

$$\boxed{\lambda(x) = 2\cos(2\pi x)}$$



$M = \int dm$

$$M = \int_0 \lambda(x)\,dx = \int_0 2\cos(2\pi x)\,dx = \frac{1}{\pi}\int 2\pi\cos(2\pi x)\,dx =$$

$$\frac{1}{\pi}\sin(2\pi x)\Big|_{x=-0.25}^{0.25} = \frac{2}{\pi}\sin(2\pi\cdot 0.25) = \boxed{\cancel{0.274}\ 0.636\ \text{kg}}$$

(make sure you are in RAD mode.)

$$x_{cm} = \frac{1}{M}\int_{x=-L/2}^{\frac{L}{2}} x\,dm = \frac{1}{M}\int_{x=-L/2}^{\frac{L}{2}} \lambda(x)\,x\,dx = \boxed{\frac{1}{M}\int_{x=-L/2}^{\frac{L}{2}} 2x\cos(2\pi x)\,dx} = ?$$

$$I = \int x^2\,dm = \int_0^{\frac{L}{2}} 2\cos(2\pi x)\,x^2\,dx = \boxed{\int_{x=-L/2}^{\frac{L}{2}} 2x^2\cos(2\pi x)\,dx}$$

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